An Aperiodic Prescribed Performance Control Scheme for Uncertain Nonlinear Systems

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Abstract— This paper proposes a low-complexity feedback control law that is updated aperiodically, in an event-triggered manner, and guarantees prescribed transient and steady state performance for uncertain nonlinear systems affine in the control. By prescribed performance, we mean that the closed-loop error trajectory converges to a predefined arbitrarily small residual set, with convergence rate no less than a certain prespecified value, having maximum overshoot less than a preassigned level. The proposed novel control design is performed in the transformed normalized error, and the triggering mechanism is extracted by guaranteed that these errors always lead to bounded closed loop signals. Moreover, the approach provides a scheme of designing and tuning the control parameters in order to achieve stabilization in a desire state in a pre-defined time $T > 0$. The efficiency of the proposed approach is verified with numerical simulations in MATLAB.

I. INTRODUCTION

It is known that many engineering applications are modeled as uncertain nonlinear dynamical systems. During the past decades, controlling such systems with feedback controllers have gain significant research attention due to important applications in automation industry, robotics, autonomous driving, system biology etc [1]–[4].

Out of the vast majority feedback laws in the literature, a promising category are the model free controllers that can guarantee predefined transient and steady state performance specification (see Fig. 1). Moreover, they have been proven a powerful strategy of controlling nonlinear systems, due to the fact that have low complexity and the controlability assumptions are the minimal [5]–[7]. The main idea of Prescribed Performance control (PPC) is to achieve through an error transformation, a control behavior such that the error signal remains within a predefined funnel as specified by user-defined time decaying functions, with both transient and steady-state characteristics to be a priori designed. However, prescribed performance control laws have usually addressed in the literature with periodic sampling. From a resource allocation perspective, periodic sampling is sometimes not preferable in that executing the control task when the system is operating satisfactorily is a waste of resources [8]–[10].

The scientific interest to event-based formulation in control, communication and signal processing has gained much attention the recent years. In the event-based systems, the

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activities are triggered by certain events instead of relying on the progression of time. This kind of formulation may be advantageous in resource-constrained applications with respect to other approaches as for example the traditional time-triggered framework [11]. An introductory paper on event-based control is [12], while the recent developments on event-based formulation are gathered in [13]. The general theoretical foundations on the event-based framework are given in [8], [14]–[16].

Motivated by the above-mentioned ideas, event-triggered prescribed performance control has been addressed recently in literature in [17], [18]. In particular, the aforementioned approaches deal with a complex analysis to derive the triggering conditions that is in some case might be not straightforward to be implemented in real platforms. In this work we propose an alternative and lower complexity approach for a class of feedback control laws that benefits from both prescribed performance ideas and the aperiodic update innovation. In particular, the proposed control law exhibits the following attributes:

- It is updated *aperiodically* only when necessary, under a novel event-triggered condition which implies that they require a smaller amount of actuation resources than periodic approaches.
- It is bounded by a known upper bound which depends on specific parameters of the dynamics of the system and the design parameters of the control in order to achieve the stabilization task.
- It can guarantee stabilization to a desired state within a predefined time $T > 0$ by tuning the control parameters appropriately, according to a methodology provided.
- It is *robust* to bounded external disturbances, uncertainties and modeling inaccuracies of the dynamic model.
- It is given in *closed form* and have *low complexity* which means that it can be directly implemented to real-time platforms.

II. NOTATION AND PRELIMINARIES

Denote by $\mathbb{R}$, $\mathbb{Q}_+$ and $\mathbb{N}$ the set of real, nonnegative rational and natural numbers including 0, respectively. $\mathbb{R}^n_{\geq 0}$ and $\mathbb{R}^n_{>0}$ are the sets of real n-vectors with all elements nonnegative and positive, respectively. Given a set S, we denote by $|S|$ its cardinality, by $S^n = S \times \cdots \times S$ its n-fold Cartesian product, and by 2^S the set of all its subsets. The notations $\|x\| := \sqrt{x^\top x}$ and $\|x\|_\infty := \max_{i \in \mathcal{N}}\{|x_i|\}$, where $\mathcal{N} := \{1, \ldots, n\}$, are used for the Euclidean and the $\infty-$norm of a vector $x \in \mathbb{R}^n$, respectively; We denote the entries of a matrix $A \in \mathbb{R}^{n \times m}$ by a_{ij}, $i, j \in \{1, \ldots, n\}$;
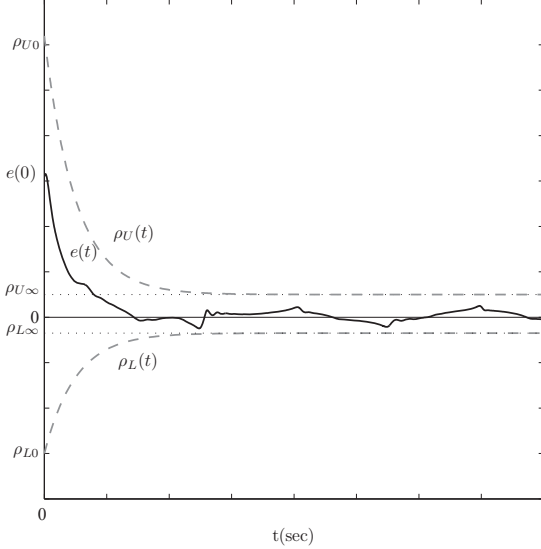
Fig. 1: Graphical illustration of the prescribed performance definition.

$I_n \in \mathbb{R}^{n \times n}$ and $0_{m \times n} \in \mathbb{R}^{m \times n}$ are the identity matrix and the $m \times n$ matrix with all entries zeros, respectively. A real matrix $A \in \mathbb{R}^{n \times n}$ is called *strictly diagonally dominant* if it holds $|a_{ii}| > \sum\limits_{j \in \mathcal{N} \setminus \{i\}} |a_{ij}|$ (see [19]).

Lemma 1: A strictly diagonally dominant matrix $A \in \mathbb{R}^{n \times n}$ is *non-singular*.

Proof: By contradiction: Let $A \in \mathbb{R}^{n \times n}$ be a strictly diagonally dominant matrix, i.e., $|a_{ii}| > \sum\limits_{j \in \mathcal{N} \setminus \{i\}} |a_{ij}|$, $\forall i \in \mathcal{N}$. Assume that A is singular, i.e., $\lambda = 0$ is one of its eigenvalues. Then, according to Gershegorins theorem (see [19]), we have that $\lambda = 0$ is located to one of the following discs:

$$|\lambda - a_{ii}| \le \sum_{j \in \mathcal{N} \setminus \{i\}} |a_{ij}| \Rightarrow |a_{ii}| \le \sum_{j \in \mathcal{N} \setminus \{i\}} |a_{ij}|,$$

for $i \in \mathcal{N}$, which is in contrast with the strictly diagonally dominance definition. $\square$

A. Prescribed Performance Control

Prescribed Performance control, originally proposed in [5], [20], refers to the methodology of designing feedback laws without any a priori knowledge of the dynamics. Moreover, the control design guarantees that a tracking error $e(t) : \mathbb{R}_{\ge 0} \to \mathbb{R}$ evolves strictly within a predefined region that is bounded by certain functions of time, achieving prescribed transient and steady state performance. The mathematical formalism is given by $-\rho(t) < e(t) < \rho(t)$, $\forall t \in \mathbb{R}_{\ge 0}$, where $\rho(t)$ is smooth and bounded decaying function of time, satisfying $\lim\limits_{t \to \infty} \rho(t) > 0$, called performance function (see Fig. 1). In particular, for the exponential performance function $\rho : \mathbb{R}_{\ge 0} \to (0, \infty)$ with $\rho(t) := (\rho_0 - \rho_\infty)e^{-\ell t} + \rho_\infty$, with ρ_0, ρ_∞, $\ell \in \mathbb{R}_{>0}$, appropriately chosen constants, $\rho_0 = \rho(0)$, is selected such that $\rho_0 > |e(0)|$ and the constants

$\rho_\infty = \lim\limits_{t \to \infty} \rho(t) < \rho_0$, represent the maximum allowable size of the tracking error $e(t)$ at steady state, which may be set to an arbitrarily small value. The latter reflects the resolution of the measurement device, thus achieving practical convergence of $e(t)$ to zero. Moreover, the decreasing rate of $\rho(t)$, which is affected by the constants ℓ in this case, introduces a lower bound on the required speed of convergence of $e(t)$. Therefore, the appropriate selection of the performance function $\rho(t)$ imposes performance characteristics on the tracking error $e(t)$.

III. SYSTEMS DYNAMICS AND ASSUMPTIONS

Consider the following *continuous time and uncertain* nonlinear dynamical system:

$$\dot{x} = f(x, \omega) + g(x)u, \tag{1}$$

where $x \in \mathbb{R}^n$ denotes the state with initial condition $x(0) \in \mathbb{R}^n$; $u \in \mathbb{R}^n$ is the control input; and $f : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \to \mathbb{R}^n$ are *unknown nonlinear vector fields*. The vector $\omega \in \mathbb{R}^n$ stands for the external disturbances, uncertainties and model mismatches and it is also assumed to be *unknown*. We introduce the following technical assumptions.

Assumption 1: The functions f, g are locally Lipschitz continuous. Moreover, there exists a positive constant $\bar{f}$ such that the following holds:

$$\|f(x, \omega)\|_\infty \le \bar{f}, \ \forall x, \omega \in \mathbb{R}^n. \tag{2}$$

Assumption 2: The matrix function g is strictly diagonally dominant with all its diagonal entries strictly positive for all $x \in \mathbb{R}^n$. Thus, there exists a strictly positive constant $\underline{g}$ such that:

$$g_{ii}(x) - \sum_{j \in \mathcal{N} \setminus \{i\}} |g_{ij}(x)| \ge \underline{g} > 0, \ \forall x \in \mathbb{R}^n, i \in \mathcal{N}, \tag{3}$$

where g_{ij}, $i, j \in \mathcal{N}$ stands for the entries of matrix g, and $\mathcal{N} := \{1, \dots, n\}$.

IV. MAIN RESULTS

In this section, we provide an event-triggered *feedback control law* which guarantee the stabilization of the system (1) in given desired states $x_{i,\text{des}}$. Define the errors along the components of the vectors $x(t)$ as:

$$e_i(t) := x_i(t) - x_{i,\text{des}}, i \in \mathcal{N}. \tag{4}$$

The performance functions are given by:

$$\rho_i(t) := (\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t} + \rho_{i,\infty}, i \in \mathcal{N}. \tag{5}$$

The goal is to design an aperiodic feedback control law u which guarantees that

$$-\rho_i(t) < e_i(t) < \rho_i(t), \forall t \in \mathbb{R}_{\ge 0}, i \in \mathcal{N}. \tag{6}$$

Define the *normalized errors* $\xi_i : \mathbb{R}_{\ge 0} \to \mathbb{R}$, as:

$$\xi_i(t) := [\rho_i(t)]^{-1} e_i(t), i \in \mathcal{N}. \tag{7}$$

Let $\{t_{i,k}\}_{k \in \mathbb{N}} > 0, i \in \mathcal{N}$, be an aperiodic sequence of sampling times for each of the components $i \in \mathcal{N}$ of the vector $x = [x_1, \dots, x_n]^\top \in \mathbb{R}^n$. Then, the following theorem provides the closed form of the *event-based feedback*

control law that guarantees , i.e., prescribed and steady state performance is achieved.

Theorem 1: Consider the system (1) satisfying Assumption 1. Given the desire state $x_{i,\text{des}}, i \in \mathcal{N}$ and a time $T > 0$. Design the control gain such that the following holds:

$$\kappa > \frac{\max\{\ell_i(\rho_{i,0} - \rho_{i,\infty})\}\mu + \overline{f}}{\mu \underline{g}}, \tag{8}$$

with $\underline{g}$ as given in (3), for every $i \in \mathcal{N}$, where $\mu \in (0,1)$ is a parameter to be appropriately chosen. Then, the event-based feedback control law $u : \mathbb{R}^n \to \mathbb{R}^n$, defined by:

$$u(x(t)) := \left[u_1(x_1(t)), \ldots, u_n(x_n(t))\right]^\top, \tag{9}$$

where:

$$u_i(x_i(t)) := -\kappa \xi_i(t_{i,k}), \forall t \in [t_{i,k}, t_{i,k+1}), i \in \mathcal{N}, \tag{10}$$

under the event-triggered mechanism:

$$t_{i,0} = 0, \tag{11a}$$

$$t_{i,k} = \inf_{t > t_{i,k}} \{|\xi_i(t)| = \mu\}, k \geq 1, \tag{11b}$$

for all the components $i \in \mathcal{N}$, guarantees the following:

- the prescribed and transient steady-state performance: $-\rho_i(t) < e_i(t) < \rho_i(t)$, $\forall i \in \mathcal{N}$, $t \in \mathbb{R}_{\geq 0}$, with ρ_i given in (5);
- Zeno behavior is excluded;
- all closed loop signals remain bounded;
- the system is driven from $x_i(0)$ to $x_{i,\text{des}}, i \in \mathcal{N}$ at precise time T.

Proof: By observing (7), it follows that if the normalized errors $\xi_i(t)$ remain in sets $(-1, 1)$ for all times, the corresponding errors $e_i(t)$ will satisfy (IV). Motivated by that, we choose to update the controller under the mechanism (11), for a desired parameter $\mu \in (0, 1)$. We will prove hereafter that the mechanism (11) and the event-based control law (9) guarantees the stability of the system with Zeno behavior excluded.

According to (11), the first triggering time $k = 0$ is at $t = 0$, for all the components $i \in \mathcal{N}$. For the rest of triggering times $k \geq 1$, we investigate the following two cases. Observing (10), (11) an upper bound of the control input, under the proposed event-triggered mechanism (11), is given by:

$$\|u(\cdot)\|_\infty \leq \kappa\mu, \text{ and } |u_i(\cdot)| \leq \kappa\mu, \forall i \in \mathcal{N}. \tag{12}$$

- $\boxed{\text{Case } 1 : \xi_i(t_{i,k}) = \mu, i \in \mathcal{N}}$. Recall that when $\xi_i(t)$ approaches the threshold 1 the system exhibits undesired behavior. When the trajectory of a component $i \in \mathcal{N}$ approaches the threshold μ, it is desired not to allow to pass this threshold, since this would lead to undesired behavior. Thus, we need to guarantee that:

$$\lim_{t \to t_{i,k}^+} \dot{\xi}_i(t) < 0 \text{ and } \dot{\xi}_i(t) < 0, \forall t \in (t_{i,k}, t_{i,k+1}), i \in \mathcal{N}. \tag{13}$$

We shall hereafter prove the aforementioned statement. By differentiating (7) with respect to time and substituting the

control law from (10) we get:

$$\rho_i(t)\dot{\xi}_i(t) = \ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t}\xi_i(t) + f_i(x(t), \omega(t)) + \Lambda_i(\xi(t)). \tag{14}$$

where the functions $\Lambda_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$ are defined by:

$$\Lambda_i(\xi(t)) := \sum_{j \in \mathcal{N}} g_{ij}(x(t))u_j(x_j(t)). \tag{15}$$

By taking the limit for $t \to t_{i,k}^+$, $i \in \mathcal{N}$ in (14), we obtain:

$$\rho_i(t_{i,k})\left[\lim_{t \to t_{i,k}^+} \dot{\xi}(t)\right] = \ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t_{i,k}}\xi_i(t_{i,k}) + f_i(x(t_{i,k}), \omega_i(t_{i,k})) + \Lambda_i(\xi(t_{i,k}^+)). \tag{16}$$

By invoking (12) and employing the property: $x \leq |x|$, $\forall x \in \mathbb{R}$, we derive a negative upper bound of the function $\Lambda_i(\xi(t_{i,k}^+))$ as follows:

$$\begin{aligned}\Lambda_i(\xi(t_{i,k}^+)) &= \sum_{j \in \mathcal{N}} g_{ij}(x(t_{i,k}^+))u_j(x_j(t_{i,k}^+)) \\ &= g_{ii}u_i(\cdot) + \sum_{j \in \mathcal{N}\setminus\{i\}} g_{ij}(\cdot)u_j(\cdot) \\ &= -g_{ii}(\cdot)\kappa\mu + \sum_{j \in \mathcal{N}\setminus\{i\}} g_{ij}(\cdot)u_j(\cdot) \\ &\leq -g_{ii}(\cdot)\kappa\mu + \sum_{j \in \mathcal{N}\setminus\{i\}} |g_{ij}(\cdot)||u_j(\cdot)| \\ &\leq -g_{ii}(\cdot)\kappa\mu + \sum_{j \in \mathcal{N}\setminus\{i\}} |g_{ij}(\cdot)|k\mu \\ &= -\kappa\left[g_{ii}(\cdot) - \sum_{j \in \mathcal{N}\setminus\{i\}} |g_{ij}(\cdot)|\right]\mu \\ &\leq -\kappa\underline{g}\mu, \tag{17}\end{aligned}$$

with $\underline{g}$ as given in (3). By observing (16) and (17) if we guarantee for every $i \in \mathcal{N}$ that:

$$\kappa\underline{g}\mu > \max\left\{\ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t_{i,k}}\mu + |f_i(x(t_{i,k}), \omega_i(t_{i,k}))|\right\},$$

or equivalently if we guarantee that: $\kappa\underline{g}\mu > \ell_i(\rho_{i,0} - \rho_{i,\infty})\mu + \overline{f}, \forall i \in \mathcal{N}$, which is equivalent to designing κ as in (8), then, we ensure that $\lim_{t \to t_{i,k}^+} \dot{\xi}_i(t) < 0$ for every $i \in \mathcal{N}$.

Define the auxiliary functions $\beta_i : \mathbb{R}_{\geq 0} \to \mathbb{R}$, $i \in \mathcal{N}$ as:

$$\beta_i(\xi(t)) := \ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t}\xi_i(t) + f_i(x(t), \omega(t)) + \Lambda_i(\xi(t_{i,k}^+)). \tag{18}$$

Due to the fact that it holds:

$$\ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t} < \ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t_{i,k}}, \\ \forall t \in (t_{i,k}, t_{i,k+1}), i \in \mathcal{N},$$

it is guaranteed that: $\beta_i(\xi(t)) < 0, \forall t \in (t_{i,k}, t_{i,k+1}), i \in \mathcal{N}$. By using the latter result in (14), we conclude that for every $i \in \mathcal{N}$ it holds that: $\dot{\xi}_i(t) < 0$, $\forall t \in (t_{i,k}, t_{i,k+1})$. Hence we have proved both statements of (13).
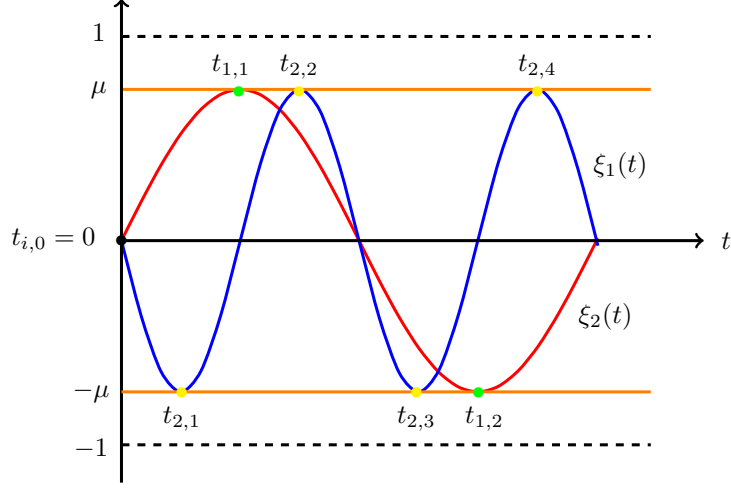
Fig. 2: Illustration of the proposed event-triggered mechanism for a dimension $n = 2$. Under the proposed event-based law (9) and the triggering mechanism (11), the signals $\xi_1(t)$ (depicted with red) and $\xi_2(t)$ (depicted with blue) evolve always in the interval $[-\mu, \mu]$ with $\mu \in (0, 1)$. The first triggering time is $t_{1,0} = t_{2,0} = 0$. The triggering times $t_{1,1}$, $t_{1,2}$ and $t_{2,1}$, $t_{2,2}$, $t_{2,3}$, $t_{2,4}$ for the signals $\xi_1(t)$ and $\xi_2(t)$ are depicted with green and yellow bullets, respectively. It holds that: $\lim\limits_{t \to t_{1,1}^+} \dot{\xi}_1(t) < 0$, $\lim\limits_{t \to t_{2,2}^+} \dot{\xi}_2(t) < 0$, $\lim\limits_{t \to t_{2,4}^+} \dot{\xi}_2(t) < 0$ and $\lim\limits_{t \to t_{2,1}^+} \dot{\xi}_2(t) < 0$, $\lim\limits_{t \to t_{2,3}^+} \dot{\xi}_2(t) < 0$, $\lim\limits_{t \to t_{1,2}^+} \dot{\xi}_1(t) < 0$. Moreover it holds that: $\dot{\xi}_1(t) < 0$, $\forall t \in (t_{1,1}, t_{1,2})$, $\dot{\xi}_2(t) < 0$, $\forall t \in (t_{2,2}, t_{2,3})$ and $\dot{\xi}_2(t) > 0$, $\forall t \in (t_{2,1}, t_{2,2}) \cup (t_{2,3}, t_{2,4})$.

- $\boxed{\text{Case } 2 : \xi_i(t_{i,k}) = -\mu, i \in \mathcal{N}}$. Recall that when $\xi_i(t)$ approaches the threshold -1 the system exhibits undesired behavior. When the trajectory of a component $i \in \mathcal{N}$ approaches the threshold $-\mu$ it is desired not to allow to pass this threshold, since this would lead to undesired behavior. Thus, we need to guarantee that:

$$\lim_{t \to t_{i,k}^+} \dot{\xi}_i(t) > 0 \text{ and } \dot{\xi}_i(t) > 0, \forall t \in (t_{i,k}, t_{i,k+1}), i \in \mathcal{N}.$$
(19)

We shall hereafter prove the aforementioned statement. By taking the limit for $t \to t_{i,k}^+$, $i \in \mathcal{N}$ in (14) and substituting $\xi_i(t_{i,k}) = -\mu$, we obtain:

$$\rho_i(t_{i,k}) \left[\lim_{t \to t_{i,k}^+} \dot{\xi}(t) \right] = -\ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t_{i,k}}\mu$$
$$+ f_i(x(t_{i,k}), \omega_i(t_{i,k})) + \Lambda_i(\xi(t_{i,k}^+)).$$
(20)

with $\Lambda_i(\cdot)$ as defined in (15). By invoking (12) and employing the property: $x \geq -|x|, \forall x \in \mathbb{R}$, we derive a positive lower bound of the function $\Lambda_i(\xi(t_{i,k}^+))$ as follows:

$$\Lambda_i(\xi(t_{i,k}^+)) = \sum_{j \in \mathcal{N}} g_{ij}(x(t_{i,k}^+))u_j(x_j(t_{i,k}^+))$$
$$= g_{ii}\kappa\mu + \sum_{j \in \mathcal{N}\setminus\{i\}} g_{ij}(\cdot)u_j(\cdot)$$
$$\geq g_{ii}\kappa\mu - \sum_{j \in \mathcal{N}\setminus\{i\}} |g_{ij}(\cdot)||u_j(\cdot)|$$
$$\geq g_{ii}\kappa\mu - \sum_{j \in \mathcal{N}\setminus\{i\}} |g_{ij}(\cdot)|k\mu$$

$$= \kappa \left[g_{ii} - \sum_{j \in \mathcal{N}\setminus\{i\}} |g_{ij}(\cdot)| \right] \mu$$
$$\geq \kappa \underline{g}\mu,$$

with $\underline{g}$ as given in (3). By observing the latter and (20), it becomes clear that if we guarantee for every $i \in \mathcal{N}$ that:

$$\kappa \underline{g}\mu >$$
$$\max \left\{ -\ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t_{i,k}}\mu + |f_i(x(t_{i,k}), \omega_i(t_{i,k}))| \right\},$$

or equivalently if we guarantee that:

$$\kappa \underline{g}\mu > \ell_i(\rho_{i,0} - \rho_{i,\infty})\mu + \overline{f}, \forall i \in \mathcal{N},$$

which is equivalent to designing κ as in (8), then we ensure that $\lim\limits_{t \to t_{i,k}^+} \dot{\xi}_i(t) > 0$ for every $i \in \mathcal{N}$. Owing to the fact that:

$$\ell_i(\rho_{i,0} - \rho_{i,\infty}) > -\ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t}$$
$$> -\ell_i(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t_{i,k}},$$
$$\forall t \in (t_{i,k}, t_{i,k+1}), i \in \mathcal{N},$$

it is guaranteed that: $\beta_i(\xi(t)) > 0, \forall t \in (t_{i,k}, t_{i,k+1})$, $i \in \mathcal{N}$, as β_i defined in (18). By using the latter result in (14), we conclude that for every $i \in \mathcal{N}$ it holds that: $\dot{\xi}_i(t) > 0$, $\forall t \in (t_{i,k}, t_{i,k+1})$. Hence we have prove both statements of (19).

Thus, by designing κ as in (8) we have ensured that $|\xi_i(t)| \leq \mu < 1$ for every $t \in \mathbb{R}_{\geq 0}$, $i \in \mathcal{N}$. The latter implies that the functions $\beta_i(t)$ are also bounded which, by invoking (14), it further implies that the signals $\dot{\xi}_i(t)$ are bounded for every $t \in \mathbb{R}_{\geq 0}$, $i \in \mathcal{N}$. To sum up, up until now we have shown that:
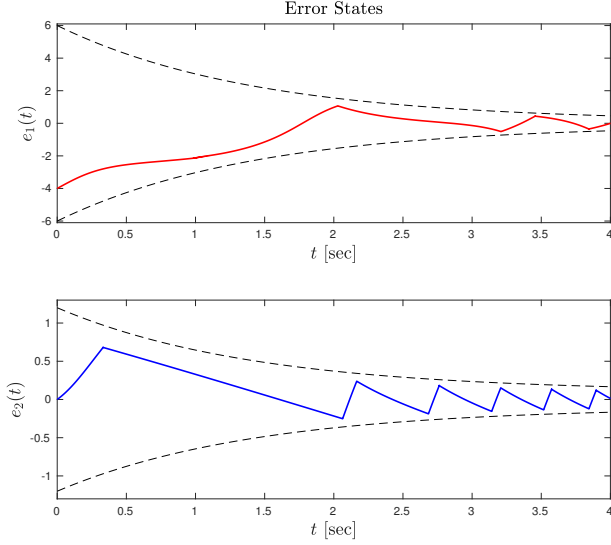
Fig. 3: The evolution of the error signals $e_1(t)$ and $e_2(t)$ as defined in (4), strictly within the funnel imposed by the performance functions $\rho_1(t)$ and $\rho_2(t)$, respectively.

- statements (13) say that if at a triggering time $t_{i,k}$ it holds that $\xi_i(t_{i,k}) = \mu$, then at the next triggering time instant $t_{i,k+1}$ it will hold that $\xi_i(t_{i,k+1}) = -\mu$;
- statements (19) say that if at a triggering time $t_{i,k}$ it holds that $\xi_i(t_{i,k}) = -\mu$, then at the next triggering time instant $t_{i,k+1}$ it will hold that $\xi_i(t_{i,k+1}) = \mu$;
- the signals $\dot{\xi}_i(t)$ are bounded for all times;
- all the closed loop signals remain bounded.

By combining the aforementioned statements, we have proven that the signals signals $\xi_i(t)$ alternate between the thresholds $-\mu$ and μ, in which the controller is updated. By using the last result and owing to the continuity of the signals $\xi_i(t)$, and the boundedness of $\dot{\xi}_i(t)$ for all the components $i \in \mathcal{N}$, there exist strictly positive constants $\underline{\tau}_i$, such that $t_{i,k+1} - t_{i,k} \geq \underline{\tau}_i > 0, \forall k \in \mathbb{N}, i \in \mathcal{N}$, which implies that Zeno behavior is excluded.

What remains to show is how we choose the convergence rates ℓ_i, $i \in \mathcal{N}$ in order to enforce the system to be driven from $x_i(0)$ to $x_{i,\mathrm{des}}$ precise in given time $T > 0$. In other words, we need to guarantee that:

$$|e_i(t)| \leq \rho_{i,\infty}, \forall t \geq T, i \in \mathcal{N}. \tag{21}$$

i.e., the system will have enter and remain in the steady state zone imposed by the constants $\rho_{i,\infty}$ (see Section II-A for more details) from time $t = T$ and onwards. By invoking (5), (7) and (11) we have that:

$$|\xi_i(t)| \leq \mu \Leftrightarrow |e_i(t)| \leq \mu\rho_i(t)$$
$$= \mu(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i t} + \mu\rho_{i,\infty}, i \in \mathcal{N}.$$

By combining the latter with (21) we have:

$$\mu(\rho_{i,0} - \rho_{i,\infty})e^{-\ell_i T} + \mu\rho_{i,\infty} \leq \rho_{i,\infty}$$
$$\Leftrightarrow e^{-\ell_i T} \leq \frac{\rho_{i,\infty}}{\rho_{i,0} - \rho_{i,\infty}} \frac{1 - \mu}{\mu}$$
$$\Leftrightarrow e^{\ell_i T} \geq \frac{\rho_{i,0} - \rho_{i,\infty}}{\rho_{i,\infty}} \frac{\mu}{1 - \mu}$$
$$\Leftrightarrow \ell_i \geq \frac{1}{T} \ln\left(\frac{\rho_{i,0} - \rho_{i,\infty}}{\rho_{i,\infty}} \frac{\mu}{1 - \mu}\right), \forall i \in \mathcal{N}. \tag{22}$$

Thus, we have proved that by choosing ℓ_i, $i \in \mathcal{N}$ as in (22), the system is driven from $x_i(0)$ to $x_{i,\mathrm{des}}$ in precise time T. This leads to the conclusion of the proof. $\qquad\square$

Remark 1: By observing (4) and Fig.1, we have that when a normalized error trajectory $\xi_i(t)$, $i \in \mathcal{N}$ approach the threshold values 1 and -1, then the error trajectory $e_i(t)$ approach the funnel imposed by the performance function $\rho_i(t)$. Taking this into consideration, the geometrical meaning of the event-triggered mechanism (11) is not to allow the normalized error trajectories $\xi_i(t)$, $\forall i \in \mathcal{N}$ to approach the values 1, -1 for all times, but keep them bounded in the interval $[-\mu, \mu]$, $\mu \in (0, 1)$ instead. This also implies that the error trajectories $e_i(t)$, $\forall i \in \mathcal{N}$ will not approach the funnel imposed by the performance functions $\rho_i(t)$ for all times. In order to intuitively understand the proposed event-triggered mechanism (11), we provide an example in $\mathbb{R}^2$ which can be depicted in Fig. 2.

V. SIMULATION RESULTS

For a simulation scenario, consider an agent with dynamics:

$$\dot{x}_1(t) = 2\cos(x_1(t) + x_2(t)) + 0.1\sin(2t) + 2u_1(t) + u_2(t),$$
$$\dot{x}_2(t) = \sin(x_1(t) + x_2(t)) + 0.1\cos(t) + u_2(t) + 2u_1(t),$$

from which it yields that: $\overline{f} = 2.1$ and $\underline{g} = 1$. We set the initial and desired states $x(0) = [0,0]^\top$, $x_{\mathrm{des}} = [4,0]^\top$, respectively. The desired time for the system to be driven from $x(0)$ to x_{des} is set to $T = 3.5\,\mathrm{sec}$. By following the procedure presented in Section IV, the proposed feedback control laws generate a unique trajectory $x(t)$, which guarantees the desired task.

In Fig. 3, Fig. 4 and Fig. 5 we depict the error states, the triggering mechanism and the control input signals, respectively, that arise for the transition of the system between $x(0)$ to x_{des}. The performance functions parameters for this transitions are chosen as: $\rho_{1,0} = 5.5$, $\rho_{0,2} = 1.2$, $\rho_{1,\infty} = \rho_{2,\infty} = 0.1$ and $\ell_1 = \ell_2 = 0.7$. Note that for this transition, the feedback controllers u_1 and u_2 are updated 5 and 11 times, respectively, in a simulation time of 4 seconds. A periodically sampled controller with sampling time of $0.1\,\mathrm{sec}$ would have been updated 40 times.

VI. CONCLUSIONS AND FUTURE RESEARCH

In this work, we have proposed a novel event-triggered mechanism for model free prescribed performance controllers for unknown and uncertain nonlinear systems. The controller has low complexity, is updated aperiodically, can guarantee stabilization in predifined time and is promising for robotic applications where time constraints are imposed to the tasks of the robot (see for example the works [21]–[23]). Future research directions include extension of this
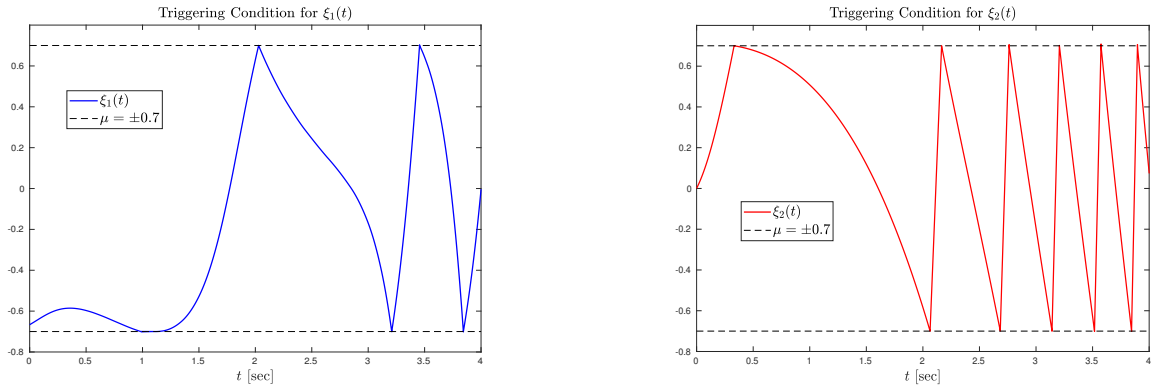
Fig. 4: The evolution of the transformed error signals $\xi_1(t)$, $\xi_2(t)$ for the under consideration simulation task.
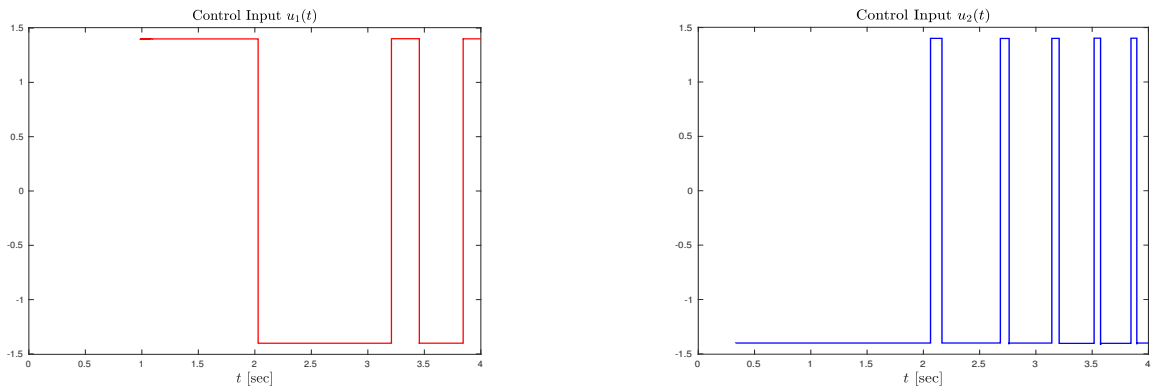

Fig. 5: The feedback control laws that guarantee the navigation of the agent between $x(0)$ and x_{des}.

framework towards a self-triggered mechanism and multi-agent setups in scenarios where the triggering mechanism also takes into account the intermittent communication between the agents.

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